PERGAMON

Expert Systems with Applications 22 (2002) 249–255

Expert Systems with Applications

www.elsevier.com/locate/eswa

Analyzing stock market tick data using piecewise nonlinear model

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Abstract

Trading in stock market indices has gained unprecedented popularity in major financial markets around the world. However, the prediction of stock price index is a very difficult problem because of the complexity of the stock market data. This study proposes stock trading model based on chaotic analysis and piecewise nonlinear model. The core component of the model is composed of four phases: The first phase determines time-lag size in input variables using chaotic analysis. The second phase detects successive change-points in the stock market data and the third phase forecasts the change-point group with backpropagation neural networks (BPNs). The final phase forecasts the output with BPN. The experimental results are encouraging and show the usefulness of the proposed model with respect to profitability.

Keywords: Stock trading; Backpropagation neural network; Chaotic analysis; Change-point detection

1. Introduction

Predicting stock market's movements is quite difficult because many factors including political events, general economic conditions, and investors' expectations influence stock markets. Previous studies on stock market prediction using artificial neural networks (ANNs) have been executed during the past decades. The earliest studies are mainly focused on applications of ANN to stock market prediction (Ahmadi, 1990; Choi, Lee, & Rhee, 1995; Kamijo & Tanigawa, 1990; Kimoto, Asakawa, Yoda, & Takeoka, 1990; Trippi & DeSieno, 1992; White, 1994). Recent research tends to hybridize several artificial intelligence (AI) techniques (Hiemstra, 1995; Tsaih, Hsu, & Lai, 1998). Some researchers included novel factors in the learning process. Kohara, Ishikawa, Fukuhara, and Nakamura (1997) incorporated prior knowledge to improve the performance of stock market prediction. In addition, Quah and Srinivasan (1999) proposed an ANN stock selection system to select stocks that are top performers from the market and to avoid selecting under performers. They concluded that the portfolio of the proposed model outperformed the portfolios of the benchmark models in terms of compounded actual returns overtime. Kim and Han (2000) proposed a genetic algorithms approach to feature discretization and the determination of connection weights for ANN to predict the stock price index. They suggested that their approach reduced the dimensionality of the feature space and enhanced prediction performance. Those studies have tended to use statistical and AI techniques in isolation. However, an integrated approach, which makes full use of statistical approaches and AI techniques, offers the promise of better performance than each method alone.

This study proposes the integrated neural network model based on the statistical change-point detection. In general, macroeconomic time series data is known to have a series of change-points since they are controlled by government's monetary policy (Mishkin, 1995; Oh & Han, 2001). The government takes intentional action to control the currency flow that has direct influence upon fundamental economic indices. For the stock price index, institutional investors play a very important role in determining its ups and downs since they are major investors in terms of marking and volume for trading stocks. They respond sensitively to such economic indices like stock price indices, the consumer price index, anticipated inflation, etc. Therefore, we can conjecture that the movement of the stock price index also has a series of change-points. In this study, we show how we have applied ANN as a nonlinear statistical modeling technique to the task of stock market index prediction, attempt to capture the significant nonlinear relationships in the indices, and reflect them into the stock trading model.

The proposed model is composed of four phases: the first phase is to determine time-lag size in input variables, the second phase is to detect successive change-points in the stock price index dataset, the third phase is to forecast the change-point group with BPN, and the final stage is to

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0957-4174/02/$ - see front matter
PII: S0957-4174(01)00058-6

forecast the output with BPN. This study then examines the predictability of the proposed stock trading model. To explore the predictability, we divided stock market data into the training data over one period and the testing data over the next period. The predictability of stock trading model is examined using three metrics.

In Section 2, we outline the development of piecewise nonlinear model and its application to the financial economics. Section 3 describes the proposed stock trading model. Sections 4 and 5 report the processes and the results of the simulated trading. Finally, the concluding remarks are presented in Section 6.

2. Prior research

2.1. Chaos analysis

Increasing evidence over the past decade indicates that stock market show chaotic behavior. A chaotic system can be modeled by a number of coupled nonlinear first-order differential equations. The minimum number of differential equations is equal to the integer that embeds the fractal dimension. The dimension of the phase space that spans the minimal number of differential equations is called the embedding dimension (Embrechts, 1994).

In addition, the level of chaos in a time series data can be characterized by a number of methods. One of the methods widely used by the physicists to test for chaos in time series data is the estimation of correlation dimension (Cecen & Erkal, 1996). The correlation dimension is an estimate of the fractal dimension and is used to differentiate between deterministic chaos and stochastic systems. It measures the correlation integral $C(\varepsilon)$, the probability that two point chosen at random will be within a certain distance of each other, and tests how the probability changes as the distance is increased (Peters, 1996).

For a given time series $\{\gamma_t : t = 1, \ldots, T\}$ of D-dimensional vectors, the correlation integral is formally defined as:

$$C(\varepsilon) = \lim_{T\to\infty} \frac{2}{T(T-1)} \sum_{i<j} I_\varepsilon(\gamma_i, \gamma_j) \quad (1)$$

where $I_\varepsilon(x, y)$ is an indicator function which is equal to one if $\|x - y\| < \varepsilon$, and zero otherwise; where $\|x - y\|$ is the norm as measured by the Euclidean distance (Wasserman, 1989).

Grassberger and Procaccia (1983) defined the correlation dimension of the time series $\{\gamma_i\}$ as follows:

$$D^m = \lim_{\varepsilon\to 0} [\log C_m(\varepsilon, T)/\log \varepsilon] \quad (2)$$

where m is embedding dimension.

2.2. Review of piecewise nonlinear model

Financial analysts and econometricians have popularly used piecewise linear models which include change-point models. They are known as models with structural breaks in economic literature. In these models, the parameters are assumed to shift—typically once—during a given sample period and the goal is to estimate two sets of parameters as well as the change-point or structural break.

These techniques have been applied to macroeconomic time series. The first study in this research area is conducted by Perron (1989, 1990) and Rappoport and Reichlin (1989). From then on, several statistics have been developed which work well in a change-point framework, all of which are considered in the context of breaking the trend variables (Banergee, Lumsdaine, & Stock, 1992; Christiano, 1992; Perron, 1995; Vogelsang & Perron, 1995; Zivot & Andrews, 1992). In those cases where only a shift in the mean is present, the statistics proposed in the papers by Perron (1990) or Perron and Vogelsang (1992) stand out. However, some variables do not show just one change-point. Rather, it is common for them to exhibit the presence of multiple change-points. Thus, it may be necessary to introduce multiple change-points in the specifications of the models. For example, Lumsdaine and Papell (1997) considered the presence of two or more change-points in the trend variables. In this study, it is assumed that the stock price indexes can have two or more change-points as well as just one change-point.

There are few artificial intelligence models to consider the change-point detection problems. Most of the previous research focused on the finding of unknown change-points for the past, not the forecast for the future (Li & Yu, 1999; Wolkenhauer & Edmunds, 1997). However, piecewise nonlinear model using structural change is known to significantly improve the performance for time series forecasting (Gorr, 1994; Oh & Han, 2001; Wasserman, 1989; White, 1994). The proposed model obtains intervals divided by change-points in the training phase, identifies them as change-point groups in the training phase, and forecasts to which group each sample is assigned in the testing phase. It will be tested whether the presence of change-points to our model may improve the predictability of stock price index.

2.3. The Pettitt test

In this study, a series of change-points will be detected by the Pettitt test (Pettitt, 1979, 1980a), a nonparametric change-point detection method, since nonparametric statistical property is a suitable match for a neural network model that is a kind of nonparametric method (Vostrikova, 1981). In addition, the Pettitt test is a kind of Mann–Whitney type statistic, which has remarkably stable distribution and provides a robust test of the change-point resistant to outliers (Pettitt, 1980b). In this point, the introduction of the Pettitt test is fairly appropriate for the analysis of chaotic time series data. The Pettitt test is explained as follows.

Consider a sequence of random variables $X_1, X_2, \ldots, X_T$, then the sequence is said to have a change-point at τ if X_t for

$t = 1, 2, \ldots, \tau$ have a common distribution function $F_1(x)$ and X_t for $t = \tau + 1, \tau + 2, \ldots, T$ have a common distribution $F_2(x)$, and $F_1(x) \neq F_2(x)$. We consider the problem of testing the null hypothesis of *no-change*, $H_0 : \tau = T$, against the alternative hypothesis of *change*, $H_A : 1 \leq \tau < T$, using a nonparametric statistic.

An appealing nonparametric test to detect a change would be to use a version of the Mann–Whitney two-sample test. Let

$$D_{ij} = \mathrm{sgn}(X_i - X_j) \tag{3}$$

where $\mathrm{sgn}(x) = 1$ if $x > 0$, 0 if $x = 0$, -1 if $x < 0$, then consider

$$U_{t,T} = \sum_{i=1}^{t} \sum_{j=t+1}^{T} D_{ij}. \tag{4}$$

The statistic $U_{t,T}$ is equivalent to a Mann–Whitney statistic for testing that the two samples $X_1, \ldots, X_t$ and $X_{t+1}, \ldots, X_T$ come from the same population. The statistic $U_{t,T}$ is then considered for values of t with $1 \leq t < T$. For the test of H_0 : no-change against H_A : change, we propose the use of the statistic

$$K_T = \max_{1 \leq t < T} |U_{t,T}|. \tag{5}$$

The limiting distribution of K_T is $\Pr \approxeq 2\exp\{-6k^2/(T^2 + T^3)\}$ for large $T \to \infty$.

The Pettitt test detects a possible change-point in the time sequence dataset. Once the change-point is detected through the test, the dataset is divided into two intervals. The intervals before and after the change-point form homogeneous groups, which take heterogeneous characteristics from each other. This process becomes a fundamental part of the binary segmentation method explained in Section 3.

3. Research design

There has been much research interest of integrating statistical techniques and neural network learning methods. It has been widely recognized that combining multiple techniques yield synergism for discovery and prediction (Gottman, 1981; Kaufman, Michalski, & Kerschberg, 1991). In this section, we discuss the architecture and the characteristics of our model to integrate the change-point detection and BPN. The BPN is applied to our model since it has been used successfully in many applications such as classification, forecasting and pattern recognition (Patterson, 1996). The BPN is used as a classification tool in Phase III and as a forecasting tool in Phase IV. Based on the Pettitt test, the proposed model consists of four phases as is explained.

3.1. Phase I (determination of time-lag size in input variables)

ANN for univariate time series forecasting is a kind of nonlinear autoregressive (AR) model and so the choice of order in the model is based on the embedding dimension of the series (i.e. stock price index) since the chaos analysis is a good method to analyze nonlinear dynamics in the time series. Nonlinear dynamics and chaos theory can provide information about input sizes (i.e. time-lags) for the design of forecasting models using neural networks (Embrechts, Cader, & Deboeck, 1994).

3.2. Phase II (construction of homogeneous groups in daily stock price index)

The Pettitt test is applied to the stock price index at time t in the training phase. The Pettitt test mentioned in Section 2 is a method for finding just one change-point in time series data. Based on this method, multiple change-points can be obtained under the binary segmentation method (Vostrikova, 1981). With H_0 as in Section 2, under the alternative hypothesis we now assume that there are R changes in the parameters, where R is a known integer. The alternative can be formulated as:

$$\begin{aligned} H_A^{(R)} &: \text{there are integers } 1 < k_1 < k_2 < \cdots < k_R \\ &< n \text{ such that } \theta_1 = \cdots = \theta_{k_1} \neq \theta_{k_1+1} = \cdots = \theta_{k_2} \\ &\neq \theta_{k_2+1} = \cdots = \theta_{k_R} \neq \theta_{k_R+1} = \cdots \\ &= \theta_n \text{ for the parameter } \theta'\text{s}. \end{aligned}$$

We note that the test statistics under the null hypothesis will remain consistent against $H_A^{(R)}$ as well, despite the fact that they were derived under the assumption that $R = 1$. Without the loss of generality, we can deduce that the tests mentioned in Section 2 are extended to the form for 'no change' against the 'R changes' alternative $H_A^{(R)}$.

Vostrikova (1981) suggested a binary segmentation method as follows. First, use the change-point detection test. If H_0 is rejected, then find $\hat{k}_1$ that is the time where Eq. (5) is satisfied. Next divide the random sample into two subsamples $\{X_i : 1 \leq i \leq \hat{k}_1\}$ and $\{X_i : \hat{k}_1 < i \leq n\}$, and test both subsamples for further changes. One continues this segmentation procedure until no subsamples contain further change-points. If exactly R changes are found, then one rejects H_0 in favor of $H_A^{(R)}$.

This process plays a role of clustering which constructs groups as well as maintains the time sequence. In this point, Phase II is distinguished from other clustering methods such as the k-means clustering method and the hierarchical clustering method that segment data samples by the Euclidean distance between cases without considering the time sequence.

3.3. Phase III (predicting the group of daily stock price index by BPN)

The significant intervals in Phase II are grouped to detect the regularities hidden in stock price index. Such groups

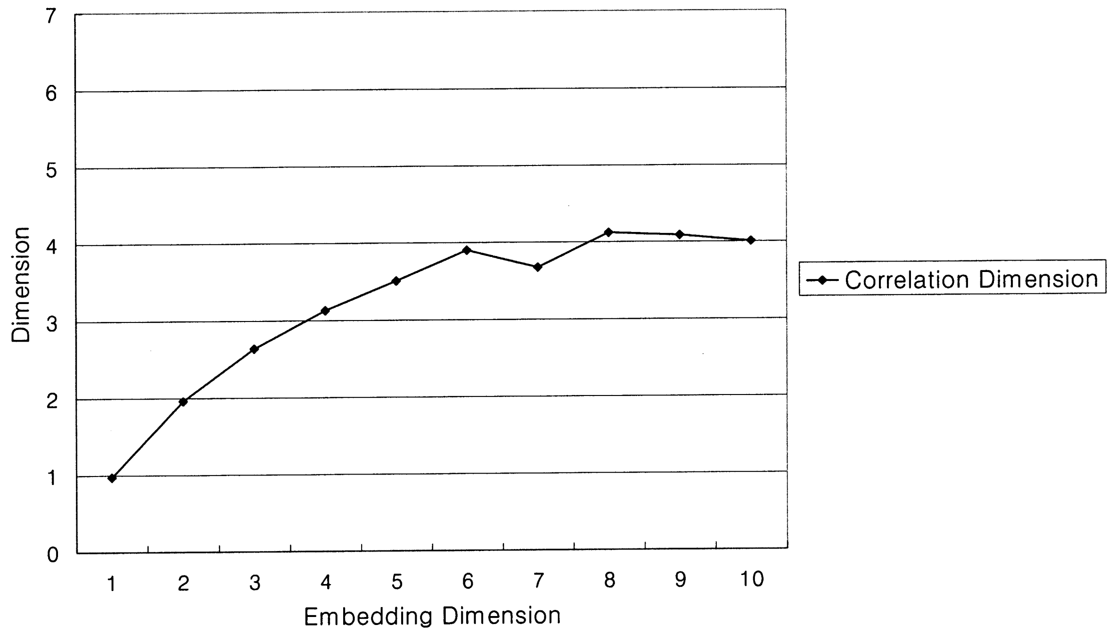

Fig. 1. Correlation dimension vs. Embedding dimension in daily KOSPI 200.

represent a set of meaningful trends encompassing stock price index. Since those trends help to find regularity among the related output values more clearly, the neural network model may have a better ability of generalization for the unknown data. In general, the error for forecasting may be reduced by making the subsampling units within groups homogeneous and the variation between groups heterogeneous (Cochran, 1977; Oh & Han, 2000). After Phase II, BPN is applied to the input data samples at time t with group outputs for $t + 1$. In this sense, Phase III is a model that is trained to find an appropriate group for each given daily stock price index.

3.4. Phase IV (forecasting the output of 1-min tick data by BPN)

After Phase III is performed, the analyzed dataset is changed from daily data to the 1-min tick data. The time-lag of input variable is kept for 1-min tick index. Phase IV is built by applying the BPN model to each group. This phase

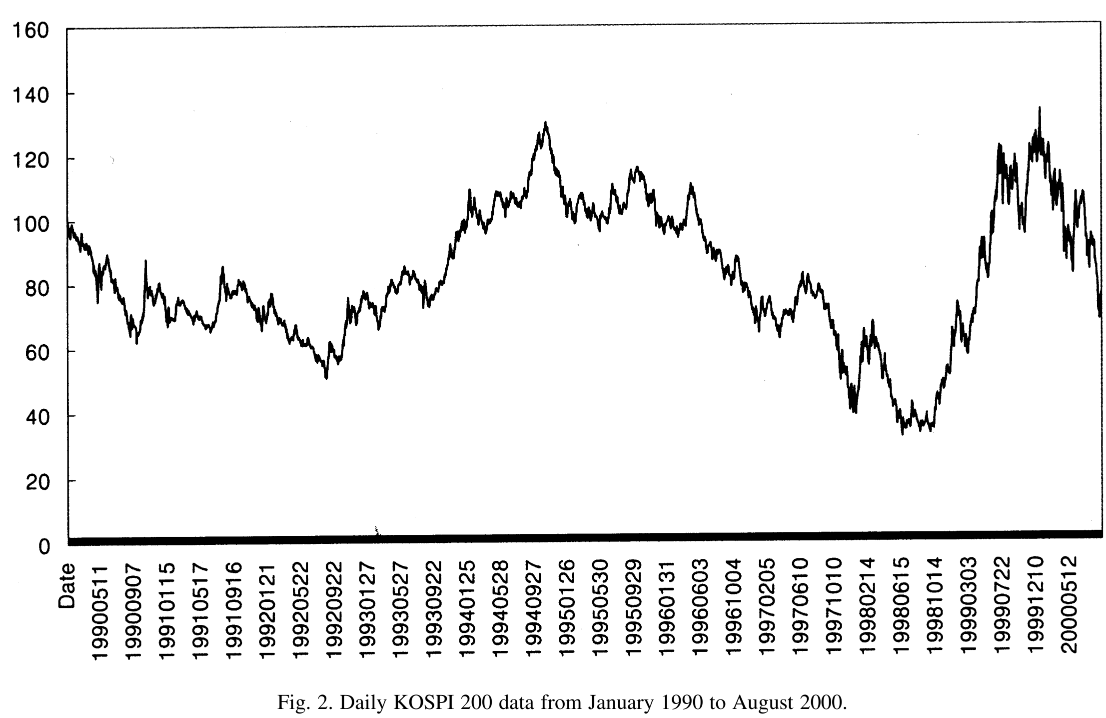

Fig. 2. Daily KOSPI 200 data from January 1990 to August 2000.

Table 1
Performance results of KOSPI forecasting based on RMSE, MAE and MAPE

Date	Model	RMSE	MAE	MAPE (%)
April 3	Basic_NN	0.5298	0.5069	0.466
	Prop_NN	0.1068	0.0788	0.073
April 6	Basic_NN	0.1842	0.1419	0.138
	Prop_NN	0.0892	0.0623	0.060
April 21	Basic_NN	1.5997	1.5970	1.662
	Prop_NN	0.5005	0.4965	0.517
May 19	Basic_NN	3.0364	3.0221	3.325
	Prop_NN	2.3052	2.2621	2.492
June 7	Basic_NN	0.2798	0.2172	0.213
	Prop_NN	0.0786	0.0598	0.058
June 13	Basic_NN	0.2055	0.1736	0.166
	Prop_NN	0.0782	0.0597	0.057
July 24	Basic_NN	1.7202	1.6998	1.780
	Prop_NN	0.6605	0.6156	0.646
July 25	Basic_NN	2.3840	2.3772	2.552
	Prop_NN	1.3341	1.3199	1.418
August 17	Basic_NN	1.9928	1.9889	2.103
	Prop_NN	0.8730	0.8642	0.914
August 28	Basic_NN	2.5951	2.5930	2.806
	Prop_NN	1.6133	1.6096	1.742
Total	Basic_NN	1.7792	1.4397	1.530
	Prop_NN	1.0480	0.7441	0.799

approximates a mapping function between the input sample and the corresponding desired output (i.e. stock price index). Once Phase IV is executed, then the sample can be used to forecast the stock price index on minute unit.

4. Experiments

Research data used in this study comes from the daily KOSPI 200 from January 1990 to August 2000. The total number of samples includes 3069 trading days. From Phase I, the results of chaos analysis in Fig. 1 indicate a saturating tendency for the correlation dimension, leading to a fractal dimension of about 5. The embedding dimension is 6. The embedding dimension of 6 indicates that 5 time-lags may be shown to a neural network to predict the 6th data point of the time series.

Table 2
Pairwise *t*-tests for the difference in residuals between the basic BPN model and the proposed models for KOSPI 200 based on the APE (***significant at 1%)

Date	*t*	*p*-value
April 3	43.868	0.000***
April 6	11.092	0.000***
April 21	186.678	0.000***
May 19	51.042	0.000***
June 7	16.696	0.000***
June 13	19.619	0.000***
July 24	140.165	0.000***
July 25	107.586	0.000***
August 17	186.991	0.000***
August 28	153.185	0.000***
Total	86.767	0.000***

The training phase involves observations from January 1990 to March 2000 while the testing phase runs from April 2000 to August 2000. The daily stock price index data is presented in Fig. 2. Fig. 2 shows that the movement of stock price index highly fluctuates.

In Phase II, the Pettitt test is applied to daily stock price index data. In this study, KOSPI 200 data is assumed to be just three change-points. This previous study demonstrated that the number of change-point on the Pettitt test does not affect the final results (Oh & Han, 2001). The study employs two neural network models. One model, labeled Basic_NN, involves five time-lag input variables to generate a forecast for $t + 1$. The second type, labeled Prop_NN, is the two-stage BPN model for four homogeneous groups. Phase III forecasts the change-point group for daily KOSPI 200 data-set. Then, Phase IV forecasts the final output for 1-min tick dataset based on the results of Phase III. For validation, 10 days (April 3, 6, 21, May 19, June 7, 13, July 24, 25 and August 17, 28) are randomly selected among testing data, April 2000–August 2000, and the earlier-mentioned two learning models are compared. The proposed model is examined on the basis of the forecasting errors and the returns from trading.

5. Results and discussions

5.1. Examining the significance of the proposed model on the forecasting errors

Numerical values for the performance metrics by the predictive model are given in Table 1. According to RMSE, MAE and MAPE, the results indicate that Prop_NN is superior to Basic_NN for all of 10 testing days.

We use the pairwise *t*-test to examine whether the differences exist in the predicted values of models according to the absolute percentage error (APE). This metric is chosen since it is commonly used (Carbone & Armstrong, 1982) and is highly robust (Armstrong & Collopy, 1992; Makridakis, 1993). Since the forecasts are not statistically independent and not always normally distributed, we compare the APEs of forecast using the pairwise *t*-test. Where sample sizes are reasonably large, this test is robust to the distribution of the data, to nonhomogeneity of variances, and to statistical dependence (Iman & Conover, 1983). Table 2 shows *t*-values and *p*-values. Prop_NN performs significantly better than Basic_NN at a 1% significant level. Therefore, Prop_NN is demonstrated to obtain improved performance using the change-point detection approach.

In summary, the trading model turns out to have a high potential in stock price index forecasting. This is attributable to the fact that it categorizes the stock price index data into homogeneous groups and extracts regularities from each homogeneous group. Therefore, Prop_NN can cope

Table 3
The profit of buying and selling simulations (US$1 = ₩1000 (approximate))

Date	Buy and hold		Basic_NN		Prop_NN	
	Point	Amount ($)	Point	Amount ($)	Point	Amount ($)
April 3	−2.20	−1100	−3.19	1595	**11.37**	**5685**
April 6	−3.15	−1575	1.52	760	**5.16**	**2580**
April 21	0.11	55	**0.78**	**390**	−0.78	390
May 19	2.42	1210	−2.43	−1215	**2.43**	**1215**
June 7	3.14	1570	−4.08	−2040	**14.88**	**7440**
June 13	−3.67	−1835	−0.73	365	**12.21**	**6105**
July 24	−3.48	1740	**3.73**	**1865**	−3.73	1865
July 25	0.75	375	−0.94	470	**0.94**	**470**
August 17	−1.06	530	**0.40**	**200**	−0.40	200
August 28	0.37	185	−0.63	−315	**0.63**	**315**
Total	−6.77	−3385	−5.57	−2785	**42.71**	**21,355**

with the noise or irregularities more efficiently than Basic_NN.

5.2. Calculating the trading returns of the proposed model

Determining stock market timing, when to buy and sell, is a very difficult problem for humans because of the complexity of the stock market. Stock market timing refers to determining the best time to buy and sell stocks, assuming that KOSPI 200 fluctuate repeatedly. To examine the profitability of Prop_NN, we apply to real 1-min tick dataset. In this study, simple trading rule is used as follows:

> Trading rules: If the predicted stock price index at $t+1$ min is greater (or less) than the stock price index at t minute, buy (or sell) the stock. The position of buying (or selling) stock should be closed 1 min later.

Trading profit earned from simulation results are summarized in Table 3. One point in KOSPI 200 futures market is worthy of 500,000 won (about 417$). The bold value in Table 3 means the best profit for a given day. Prop_NN obtains the best profit point for 7 days of 10 days. Prop_NN earnes total profit points in 10 testing days amount to 42.71 point (about 18,000$), which is superior to that of Buy and hold strategy and Basic_NN with minus fee for trade. Therefore, Prop_NN provides high profit in real time stock price index trading.

6. Concluding remarks

This study has suggested the stock trading model based on chaotic analysis and piecewise nonlinear model. The proposed model consisted of four phases. The first phase selects the time-lag size for input variable based on the chaos theory. The second phase conducts the nonparametric statistical test to construct the homogeneous groups. The third phase applies BPN to forecast the change-point group in the third phase. The final phase applies BPN to forecast the output.

The proposed trading model using piecewise nonlinear model performs significantly better than the basic BPN model at a 1% significant level. These experimental results imply the change-point detection has a high potential to improve the performance. In conclusion, we have shown that Prop_NN improves the predictability of stock price index significantly. Through the trading, furthermore, Prop_NN also demonstrates to get high profit.

Prop_NN has the promising possibility of improving the profit if further studies are to focus on various trading rules and strategies. In Phase IV, other intelligent techniques besides BPN can be used to forecast the output. In addition, the proposed model may be applied to other chaotic time series data such as interest rate and exchange rate prediction.

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